

Understanding Polarization Correlation of Entangled Vector Meson Pairs

Xun Chen, Siguang Wang**, Yajun Mao*

*School of Physics and State Key Laboratory of Nuclear Physics and Technology
Peking University, Beijing 100871, China*

February 17, 2012

Quantum mechanics (QM) disfavours local realism because it violates Bell's theorem [1,2], a theorem that must be valid for all local hidden variable theories (LHVT). Experimental tests of local realism in high energy particle physics have been proposed and carried out [3–5], but no decisive conclusion has been drawn [6]. We find that the correlated distribution between polarization vectors of two entangled mesons predicted by QM can be reproduced in a natural way, by interpreting the two-body decay of vector mesons as a measurement to their polarization vector. The interpretation provides a method to simulate the correlated decay of entangled vector meson pairs, and a potential experimental approach for the discrimination between QM and LHVT in particle physics. Related data analysis can be carried out at currently running experiments and could give deeper insight for the understanding of local realism.

Debates over the interpretation of QM have lasted for decades. In their famous paper in 1935 [7], Einstein, Podolsky and Rosen (EPR) argued that QM is incomplete by considering an *Gedankenexperiment*, which is now known as the EPR paradox. The paradox challenges the principle of uncertainty that the position and momentum of a particle can't be precisely measured simultaneously. This paradox also extends to other pair of conjugate physics quantities. In 1951, Bohm expressed the paradox with particle spins in his book [8] and considered a set of alternative theories retaining locality and reality, which are called LHVT [9]. It had been considered that one of the LHVT may replace QM, but Bell concluded that no LHVT can reproduce all the predictions of QM [1,2], since the predictions of LHVT would satisfy the Bell's inequality while those of QM violate it. Other forms of inequalities similar to the original Bell's inequality are derived [10,11] for the experimental discrimination of the LHVT and QM.

Since 1972, a series of EPR experiments have been carried out using the entangle photon pairs [12–16], and their results favored QM. But conclusions can not be drawn decisively because these experiments could not be treated loophole free [17]. Tests of the Bell's inequality in particle physics have also been considered. Back to the 1960's, the entanglement of $K^0\bar{K}^0$ pair was noticed and studied [3]. Violation of a Bell's inequality have been observed in the $B^0\bar{B}^0$ mesons from $\Upsilon(4S)$ decay by the Belle collaboration [4,5]. But it was argued that the inequalities used is not a genuine Bell's inequality so that it can't make a discrimination between QM and the local realism theories [6]. Törnqvist suggested that the weakly decay particles, such as the $\Lambda\bar{\Lambda}$ pairs generated from the decay of J/ψ or η_c , could be used for the test of nonlocality of the predictions of QM by assuming the CP invariance [18]. The DM2 collaboration has studied this with about 10^3 $J/\psi \rightarrow \Lambda\bar{\Lambda}$ events [19], but the statistics is not sufficient to give a decisive conclusion. Test of the local realism with the vector meson pairs from the pseudoscalar cascade decays, such as $\eta_c \rightarrow VV \rightarrow (PP)(PP)$, was also suggested [20,21].

*maoyj@pku.edu.cn, **siguang@pku.edu.cn

In this letter, we try to provide a straightforward understanding of the polarization correlation of entangled vector meson pairs. In η_c 's rest frame, the cascade decay process of η_c to vector mesons $V_1 V_2$ is illustrated in Fig. 1. The z -axis is chosen to be parallel to meson V_1 's momentum. The two vector mesons form an entangled state, and their polarization vectors are not fixed. After the decay of meson V_1 , its polarization is given by the momentum projection of its daughter particle d_1 on the x - y plane. Another daughter particle of V_1 is not illustrated as it has the opposite momentum component on the x - y plane to that of d_1 . The meson V_2 's polarization is determined in the same way.

For the entangled state of two vector mesons, the total spin is $S = 1$ according to the conservation of the C and P parity. In the situation that the total spin projection s on the z -axis is 0, both mesons are transversely polarized, and the wave function of the system is [20, 21]

$$|\Psi\rangle = \frac{1}{\sqrt{2}} (|1\rangle_z |-1\rangle_z - |-1\rangle_z |1\rangle_z), \quad (1)$$

where the subscript z means the z -axis. It can also be expressed in terms of bases in the x - y plane as

$$|\Psi\rangle = \frac{1}{\sqrt{2}} (|0\rangle_{\alpha_\perp} |0\rangle_\alpha - |0\rangle_\alpha |0\rangle_{\alpha_\perp}), \quad (2)$$

where the subscript α means an arbitrary axis in the x - y plane, and α_\perp indicates another axis on the same plane and perpendicular to α .

The correlated distribution between the polarization vectors of the mesons is the same as the probability that the spin of one meson has zero component along arbitrary axis α_1 and another meson has zero component along arbitrary axis α_2 . It can be calculated directly with QM. The result is given in Ref. [20] and reads,

$$\begin{aligned} P(m_{\alpha_1} = 0, m_{\alpha_2} = 0) &= |\langle \alpha_1 | 0 \rangle_{\alpha_2} \langle 0 | \Psi \rangle|^2 \\ &\propto \frac{1}{2} \sin^2(\alpha_1 - \alpha_2) \\ &= \frac{1}{2} \sin^2 \Delta\alpha, \end{aligned} \quad (3)$$

The same form of distribution can be obtained in another way with the understanding of the entanglement of polarization. When one meson decays, its polarization vector is determined and has an azimuthal angle α . According to Eq. 2, the polarization vector of the second meson should have the azimuthal angle of $\alpha \pm \pi/2$. That is similar to the case that when we measure the spin of one of the entangled spin-1/2 particles in one direction, another one should have a definite spin value in the same direction. However, the polarization vector of the second meson is finally determined by measurement, i.e. its own decay.

The relation between the polarization vectors and the decays of vector mesons are also illustrated in Fig. 1. The polarization vector \mathbf{P}_1 of meson V_1 lays on the projection of the meson's daughter particle's momentum on the x - y plane. We denote the axis with the same direction as X , which is the α axis in Eq. 2. The α_\perp axis of V_2 is denoted as Y , which is perpendicular to both the z and X axis. The projection of the momentum vector of V_2 's daughter particle on the x - y plane is denoted as \mathbf{P}_2 . The angle $\Delta\alpha$ between X and \mathbf{P}_2 is exactly the same angle between the polarization vectors \mathbf{P}_1 and \mathbf{P}_2 . As both the vectors are perpendicular to the z -axis, the angle is also equal to the angle between the decay planes of the two vector mesons.

It is obvious that the probability distribution of angle $\Delta\alpha$ can be calculated if the angular distribution of \mathbf{P}_2 is known. The angular distribution of the final state particles in the decay can be described by the helicity formalism [22], which is widely used in particle physics. For the two-body decay, as the two final state particles are back-to-back in their center-of-mass frame, their directions can be characterized by the polar angle θ and azimuthal angle φ . The

cross section of a spin-1 vector meson decay into two spin-0 particles can be written down as

$$\frac{d\sigma(\theta, \varphi)}{d\Omega} = \frac{d\sigma}{d\cos\theta d\varphi} = \frac{3}{4\pi} |D_{M0}^1(\varphi, \theta, -\varphi) A|^2 \propto |Y_1^M(\theta, \varphi)|^2, \quad (4)$$

where D_{M0}^1 is the Wigner D-Matrix $D_{m\lambda}^j$ with spin $j = 1$, and spin-projection $m = M$ and $\lambda = 0$, and A is a matrix element without any angular dependence. $Y_1^M(\theta, \varphi)$ is the spherical harmonic function with $l = 1$. M depends on the selection of spin quantization axis. For the decay of V_2 , the spin quantization axis is chosen to be in the x - y plane with $m = 0$, which is the Y -axis, so the cross section has an angular dependence of $|Y_1^0(\theta', \varphi')|^2 \propto \cos^2 \theta'$, where the polar angle θ' is defined as the angle between the momentum and the Y -axis, and the azimuthal angle φ' is defined in the X - z plane.

The probability distribution should be expressed with θ and φ in the original xyz coordinate frame. It can be obtained with a simple transformation:

$$P(\cos\theta, \varphi) = P(\cos\theta', \varphi') \left| \frac{\partial(\cos\theta', \varphi')}{\partial(\cos\theta, \varphi)} \right| = P(\cos\theta', \varphi'), \quad (5)$$

where the Jacobian determinant is equal to 1 due to the rotation invariance. Then the probability of \mathbf{P}_1 and \mathbf{P}_2 have an angle $\Delta\alpha$ can be obtained by integrating the angular distribution above over θ :

$$P(\Delta\alpha) = \int_{-1}^1 P(\cos\theta, \varphi) d\cos\theta \cdot \left| \frac{\partial\varphi}{\partial\Delta\alpha} \right| = \int_{-1}^1 P(\cos\theta, \varphi) d\cos\theta, \quad (6)$$

where $\Delta\alpha$ and φ differ by a constant, which depends on the selection of the coordinate system.

It is clear that the value of $P(\Delta\alpha)$ is irrelevant to the direction of the Y -axis. To simplify the calculation, the Y -axis is renamed z' -axis, and chosen to coincide with the x -axis without loss of generality. The y' -axis is the same as the y -axis and the x' -axis is anti-parallel to the z -axis. With this definition, the relations between angles (θ, φ) and (θ', φ') can be written down:

$$\begin{aligned} \cos\varphi' \sin\theta' &= -\cos\theta, \\ \sin\varphi' \sin\theta' &= \sin\varphi \sin\theta, \\ \cos\theta' &= \cos\varphi \sin\theta. \end{aligned} \quad (7)$$

So that the angular distribution is given by

$$P(\cos\theta, \varphi) \propto \sin^2\theta \cos^2\varphi. \quad (8)$$

This definition also gives $\varphi = \Delta\alpha - \pi/2$, so that the distribution between the polarization vectors is

$$P(\Delta\alpha) \propto \cos^2\varphi = \sin^2\Delta\alpha. \quad (9)$$

Eq. 9 gives the same form of result as that given in Eq. 3. The prediction on the correlated distribution of QM is reproduced naturally.

The calculation above provides a method to simulate the quantum entanglement in the process of cascade decay of $\eta_c \rightarrow VV \rightarrow (PP)(PP)$. The key of the simulation is to sample the directions of final state particles. That is straightforward from Fig. 1. At first, either of the two vector mesons is selected and denoted as V_1 . In the rest frame of η_c , the decay of V_1 would follow the angular distribution of $|Y_1^{\pm 1}(\theta, \varphi)|^2 \propto \sin^2\theta$. The angles θ and φ can be sampled from this distribution. The sampled φ determines the quantization axis of the second vector meson V_2 . The decay of

V_2 follows the angular distribution of $|Y_1^0(\theta', \varphi')|^2 \propto \cos^2 \theta'$, from which the angles θ' and φ' are sampled. By rotating the unit vector constructed with the sampled θ' and φ' back into the same coordinate frame of V_1 , the decay angle θ and φ of V_2 can be obtained.

This method is implemented for the process of $\eta_c \rightarrow \phi\phi \rightarrow K^+K^-K^+K^-$. To check whether the simulation method works as expected, 10,000 events are generated, and the correlated distribution $f(\Delta\alpha)$ is extracted. The result is plotted in Fig. 2. The distribution is fitted with the function of $p \cdot \sin^2 \Delta\alpha$. It is clear that the distribution satisfies that given in Eq. 3 and Eq. 9. The method works well.

Another more important conclusion of the understanding is that it provides a potential approach to discriminate QM and LHVT in high energy experiments. An analogy between the decay and the polarimeter used in the optical experiments to test the Bell inequalities could be made. Though the direction of the “polarimeter” can’t be selected at will, high statistical data make the revelation of the correlated distribution of the polarization vectors possible. On the contrary, it is difficult to obtain such correlated distributions in the optical experiments, because one need to place the two polarimeters in all possible direction and try all the combination of directions. A variation of the CHSH inequality has been derived in Refs. [20,21]. The inequality is violated by QM at some points due to the special correlated distribution predicted by QM. In other words, if the predicted correlated distribution is observed, one could say definitely that the inequality would be violated, and QM would be favored.

It is possible to measure such a correlated distribution in current high energy experiments. The branching fraction of $J/\psi \rightarrow \gamma\eta_c$ is about $1.7 \pm 0.4\%$, of $\eta_c \rightarrow \phi\phi$ is about $2.7 \pm 0.9 \times 10^{-3}$ and $\phi \rightarrow K^+K^-$ is about $48.9 \pm 0.5\%$ [23]. At the BES-III experiments, $10^9 J/\psi$ particles are expected to be recorded in the next few years [24]. About 10^5 events for the $\eta_c \rightarrow \phi\phi \rightarrow K^+K^-K^+K^-$ process are expected to be collected. The statistics would be sufficient for the measurement of the correlated distribution. The analysis result might be of fundamental importance.

In summary, with the understanding of the polarization correlation, we reproduce the same form of correlated distribution predicted by QM of the polarization vectors of entangled vector meson pairs. This provides a way for the simulation of the correlation in the pseudoscalar cascade decay. The measured correlated distribution in high energy particle experiments can be used to make a discrimination between QM and LHVT. The measurement is possible in existing particle physics experiments.

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Acknowledgment

The authors thank Prof. Cong-Feng Qiao, Dr. Junli Li, Dr. Peng Sun and Dr. Jian-Hui Zhang for the valuable discussions. The authors also would like to thank Mingsu Cong, Zijun Xu, Zhongming Qu, Jiangbo Wei for the related former research. This work is supported by the Postdoctoral foundation of Peking University, and the National Natural Science Foundation of China (10979010, 10835001).

Author Contributions Y. Mao suggested the original idea. S. Wang performed the simulation. X. Chen did the derivation and calculation, and wrote the manuscript. All authors discussed the manuscript.

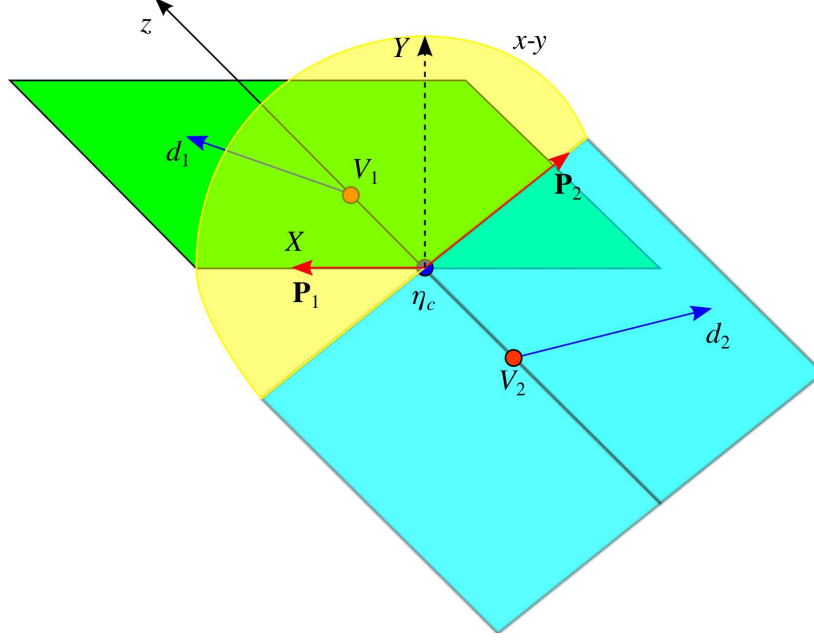


Figure 1: Decay of a pair of entangled vector mesons from the decay of η_c . \mathbf{P}_1 and \mathbf{P}_2 are the polarization vector of meson V_1 and V_2 , respectively.

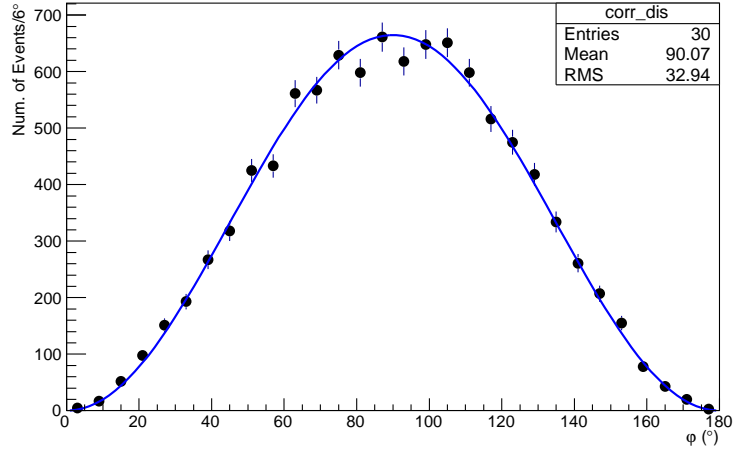


Figure 2: The correlated distribution of the polarization vectors of the two vector mesons. The x -axis is the angle between the polarization vectors. The distribution follows the formula of $f(\Delta\alpha) \propto \sin^2 \Delta\alpha$.